

## INVERSION OF TRANSIENT EDDY-CURRENT SIGNALS FOR THE DETERMINATION OF CONDUCTING PLATE PARAMETERS

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### INTRODUCTION

In traditional eddy-current NDE, an electromagnetic field is excited by a coil driven with a time-harmonic current. Information about the test piece is found from the effect of induced current on the coil impedance. Additional information may be obtained by using a number of different frequencies. In contrast, transient eddy-currents are excited by means of a pulsed coil current and perturbations of the magnetic field due to the test piece are monitored by observing the variation of the induced emf across the coil with time. Alternatively, the field perturbations may be detected by sensing the magnetic field using a Hall device.

A transient eddy-current excitation has the advantage that the resulting field consists of an entire spectrum of frequencies. A sampled transient signal can consist of many hundreds of data points which means that it is possible to extract more information than could be found from a few time harmonic measurements. The initial part of the transient response contains information about the surface features of the specimen while the latter signal includes effects of interactions that take place deeper within the sample. For example the signals due to a half-space and a plate of the same conductivity are similar up to a certain point in time when the effects of the back surface of the plate are observed.

In the frequency domain, the magnetic field due to a current carrying filament above a half-space conductor is given by the equations derived by Hammond[1]. The superposition of these solutions gives the field due to a coil of rectangular cross section[2]. For a half-space conductor, the frequency domain solution may be transformed into the time domain analytically but for a general stratified conductor, the transformation must be done numerically, for instance using the fast Fourier transform (FFT). The time domain probe signals may be expressed in terms of the electromagnetic field and the predictions of the theory compared with experimental measurements.

Simple parametric inversions may be performed on experimental results to yield estimates of plate conductivity and probe liftoff. This process can be extended to multi-layer plate-air systems where layer thicknesses are sought. In order to invert transient measurements made on a parallel plate arrangement, the probe signal is computed at discrete intervals in time and discrete intervals in the parameter space. These predictions are stored in a database and accessed by a parameter inversion scheme. The approach means computer intensive calculations are performed only once to establish the database. The search of the database then takes place at a speed that provides the sample parameters in real time.

## THEORY

Consider eddy-currents excited in a sample by means of a current carrying coil of rectangular cross-section with its axis normal to the sample surface. The z-component of the magnetic field in the axial direction is given by[3]

$$H_z(\rho, z, t) = \frac{1}{2} \int_0^\infty \Psi(\kappa, t) e^{-\kappa z} \mathcal{J}(\kappa) J_0(\kappa, \rho) d\kappa, \quad (1)$$

where  $\rho$  and  $z$  are cylindrical polar coordinates defined with respect to the coil axis. The coil function  $\mathcal{J}(\kappa)$ , is given by[4]

$$\mathcal{J}(\kappa) = \frac{2n}{\kappa} e^{-\kappa h} \sinh(b\kappa) [a_1^2 \mathcal{X}(a_1 \kappa) - a_2^2 \mathcal{X}(a_2 \kappa)]. \quad (2)$$

Here  $a_1$  is the coil outer radius and  $a_2$  the inner radius.  $h$  is the height of the center of the coil and its length is  $2b$ .  $\mathcal{X}$  is defined as[4]

$$\mathcal{X}(\alpha) = \frac{\pi}{2\alpha} [J_1(\alpha) \mathcal{H}_0(\alpha) - J_0(\alpha) \mathcal{H}_1(\alpha)], \quad (3)$$

where  $\mathcal{H}_0$  and  $\mathcal{H}_1$  are Struve functions[5].

The function  $\Psi$  in equation 1 depends on the reflection from the sample and the time variation of the excitation current. It may be evaluated analytically for a half-space, or numerically for multi-layered systems. For a half-space, with zero rise time excitation,  $\Psi(\kappa, t) = \Phi(\kappa, t)$ , where[3]

$$\Phi(\kappa, t) = 2I_0 \left[ \sqrt{\frac{t}{\pi\tau}} e^{-t/\tau} + \frac{1}{2} \left( 1 + \frac{2t}{\tau} \right) \text{erf} \left( \sqrt{\frac{t}{\tau}} \right) - \frac{t}{\tau} \right] - I_0, \quad (4)$$

where  $\tau = \mu_0 \sigma / \kappa^2$ . For a current excitation given by

$$I(t) = I_0 (1 - e^{-t/\tau_0}), \quad (5)$$

the function  $\Psi(\kappa, t)$  has the form

$$\Psi(\kappa, t) = \Phi(\kappa, t) - \tau_0 \Phi'_0(\kappa, t), \quad (6)$$

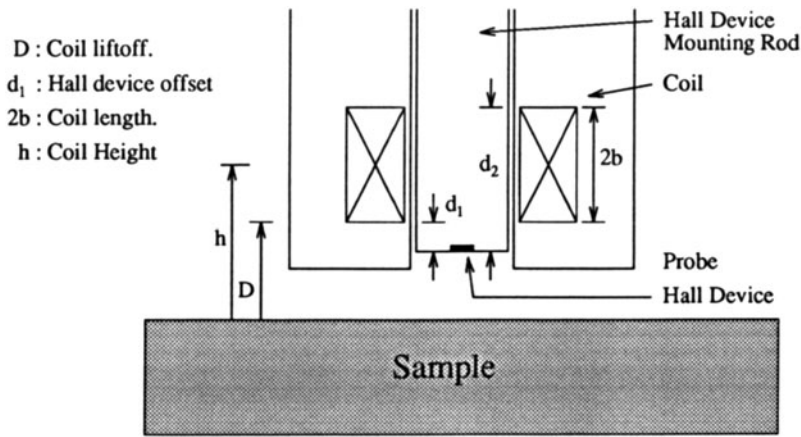


Figure 1 : Probe parameters.

where

$$\Phi'_0(\kappa, t) = \frac{2I_0}{\tau_0(1 - \nu\tau)} \left\{ e^{-t/\tau_0} \left[ 1 - \sqrt{\nu\tau} \operatorname{erf}(\sqrt{\nu t}) \right] - \operatorname{erfc} \left( \sqrt{\frac{t}{\tau}} \right) \right\} - I_0 \frac{1}{\tau_0} e^{-t/\tau_0}. \quad (7)$$

The magnetic field given by Equation (1) has been calculated using  $\Psi(\kappa, t)$  given by Equations (6), (4) and (7). These results are compared with experiment below.

## EXPERIMENTAL PROCEDURE

In the experimental system, eddy currents are induced by means of a normal cylindrical coil of rectangular cross section. The magnetic field is measured using a Hall device located on the coil axis (Figure 1), 0.15mm below the base of the coil. The important probe parameters are shown in Table 1.

The probe coil is driven by a bipolar square wave current source with exponentially rising and falling edges (Equation 5) at a repetition rate of approximately 12Hz. The rise

Table 1 : Eddy current probe parameters.

Probe Parameter	Value
Inner Radius, $a_2$	7.10mm
Outer Radius, $a_1$	11.41mm
Coil Liftoff, $D$	0.60mm
Hall Offset, $d_1$	0.15mm
Coil Length, $2b$	5.00mm
Number Of Turns	2550
Wire Gauge	46 S.W.G.
Inductance, $L_c$ (1kHz)	121.79mH
Series Resistance, $R_c$	606.26 $\Omega$
Time constant ( $L_c/R_c$ )	205.5 $\mu$ sec
Resonant Frequency	40.4kHz

time  $\tau_0$ , referred to as the system time constant, is  $275\mu\text{secs}$ . To minimize the effects of temperature variations on the coil and circuit, a transconductance amplifier is used to drive the coil. In this way the coil current is always a fixed function of the transconductance amplifier's input voltage. Magnetic field measurements are made by sampling the Hall device outputs at  $20\mu\text{sec}$  intervals using an analog to digital converter card installed in a personal computer (PC).

Typically, twenty transient repetitions are recorded by the PC and averaged to give a mean value. The negative portion of the averaged transient is subtracted from the positive and the result is multiplied by a calibration factor to give the magnetic field at the Hall sensor. The Hall device sensitivity is not known, although it could be measured along with the gains in the various signal amplification stages to find the required calibration factor. However, the sensitivity of the Hall sensor and the amplification are temperature dependant, therefore it is advantageous to repeat the calibration frequently as conditions change. This is done continually using the asymptotic value of the signal near the end of the transient tail. This value is independent of the workpiece and therefore is the same as with the probe in free space. When the probe is in free space, the field on the axis is given by a simple formula stated below (Equation 8). Hence it is possible to deduce the factor converting transient voltages to magnetic field values. The field at the Hall device in air is given by

$$H_z = K d_2 \ln \left[ \frac{a_2 + \sqrt{a_1^2 + d_2^2}}{a_1 + \sqrt{a_2^2 + d_1^2}} \right] - K d_1 \ln \left[ \frac{a_2 + \sqrt{a_1^2 + d_2^2}}{a_1 + \sqrt{a_2^2 + d_1^2}} \right], \quad (8)$$

where

$$K = \frac{N I_0}{2(a_1 - a_2)(d_2 - d_1)}. \quad (9)$$

Here  $a_2$  is the inner radius and  $a_1$  the outer radius.  $d_1$  and  $d_2$  are the distances between Hall device and the bottom and top of the coil respectively.  $N$  is the number of turns.  $I_0$  is the peak coil current. Peak coil current is monitored and Equation 8 evaluated to yield the true peak magnetic field.

## COMPARISON OF THEORY WITH EXPERIMENT

The magnetic field at the position of the Hall sensor has been calculated using Equation 1 for a range of experimental conditions. Results have been obtained for thick aluminum and copper specimens of conductivities 38.8% IACS and 103.3% IACS respectively. The conductivities have been measured using a commercial test instrument. The magnetic field in air is subtracted from both experimental and predicted transients, therefore the resultant signal arises solely from the specimen. The experiments are carried out with a number of probe liftoff values, 0.00, 0.10, 0.25, 0.36 and 0.84mm. Figure 2 compares theory and experiment for the aluminum sample modelled as a half-space. Similar results are found for the copper using a conductivity of 100% IACS.

## LAYER INVERSION

The transient response has been predicted at equal intervals over a range of conductivities and liftoffs. The readings are recorded at sample points spaced at regular intervals in time. Twenty samples of each transient signal at  $500\mu\text{sec}$ . intervals are used for the inversion scheme. The sample points are at  $t_0 = 0\mu\text{secs.}$ ,  $t_1 = 500\mu\text{secs.}$ ,  $t_2 = 1000\mu\text{secs.}$ , ...  $t_{19} = 9500\mu\text{secs.}$  The values of the magnetic field at these points are stored in a database together with the corresponding values of conductivity and liftoff.

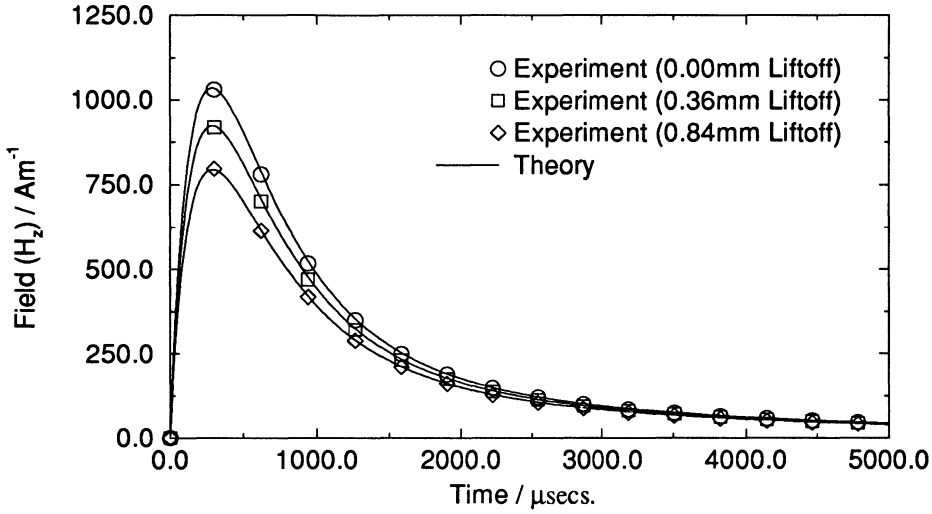


Figure 2 : Half space transients - predictions and experiment (38.8% IACS aluminum).

An error function is generated which is a measure of the disagreement between the experimental measurements and the predictions for a particular parameter set. In general, this function is given by

$$\mathcal{E}(\bar{p}) = \sum_{i=0,n} |H_{t_i}^{(exp)} - H_{t_i}(\bar{p})|, \quad (10)$$

where  $\bar{p}$  is a vector whose components are the parameters that are sought.  $t_i$  is the time at which sample  $i$  is taken.  $H_{t_i}(\bar{p})$  is the predicted magnetic field at the Hall sensor and  $H_{t_i}^{(exp)}$  the experimentally measured field. The parameter set that is required is the one that minimizes the error function. For example, in a conductivity measurement that corrects for liftoff, suitable conductivity and liftoff values are sought which yield a predicted transient closely matching the measured one.

As a check of functionality, an inversion has been carried out using the data given above for the aluminum and copper conductors to estimate conductivity and liftoff.

For the aluminum, predictions of the transient signals are made at conductivities between 30% and 50% IACS, at 0.25% IACS intervals and for liftoff values in the range 0.00mm to 1.00mm, at 0.05mm intervals. These predictions are stored in the database. Table 2 shows the optimum liftoff and conductivity following the database search and compares these values with direct measurement. For the copper, predictions of the transient signals are made at conductivities in the range 90% to 110% IACS, at 0.25% IACS intervals and for liftoff values in the range 0.00mm to 1.00mm, at 0.05mm intervals. Table 3 shows the optimum liftoff and conductivity compared with measured values.

Parameters of layered systems have been obtained in a similar way. A database of signal predictions for tabulated values of the required parameters is generated. A data set is then sought which minimizes the difference between the predictions and the experimental results. The parameters corresponding to this data set are optimum in the sense that they minimize the error function. Figure 3 shows the layer system under investigation. To

Table 2 : Measured and predicted half-space conductivity and probe liftoff for aluminum (38.8% IACS).

Measured	Predictions	
Liftoff / mm	Liftoff / mm	Conductivity %IACS
0.00	0.00mm	38.00
0.10	0.10mm	37.75
0.25	0.25mm	37.75
0.36	0.35mm	37.75
0.84	0.80mm	37.75

Table 3 : Measured and predicted half-space conductivity and probe liftoff for copper (103.3% IACS).

Measured	Predictions	
Liftoff / mm	Liftoff / mm	Conductivity %IACS
0.00	0.00mm	100.25
0.10	0.10mm	100.25
0.25	0.30mm	101.00
0.36	0.35mm	99.50
0.84	0.85mm	101.00

provide a reference, measurements are made with the two plates in contact. The transient response obtained over the contacting plates is subtracted from the responses over the other arrangements shown.

In order to restrict the size of the database and the computational burden, the minimum error is sought in two stages. Firstly, a coarse search is carried out to obtain approximate layer dimensions. Then a search on a finer grid is performed to determine the layer heights to within  $\pm 0.1\text{mm}$ . The results are summarized in Tables 4, 5 and 6.

## CONCLUSIONS

The inversion scheme described here provides a rapid method for the evaluation of either half-space conductivity and liftoff or the dimensions of a four-layered stratified system of conductors where conductivity and liftoff are known. The former procedure is ideally suited to conductivity measurement of metals with non-conducting coatings or the thickness measurements of such coatings. The latter, although requiring considerably more development could find applications in the measurement of corrosion between plate joints or for evaluating the thickness of conducting coatings on conducting substrates.

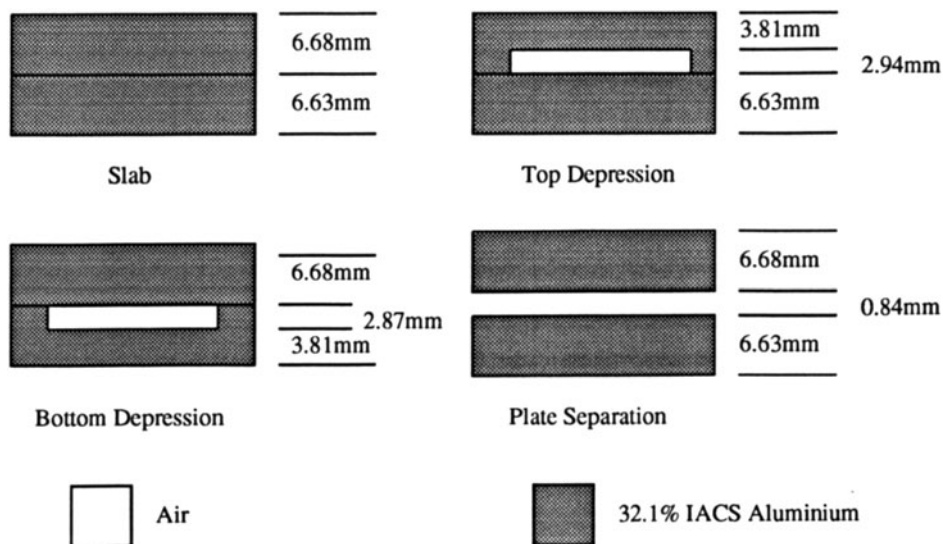


Figure 3 : Plate arrangements used to simulate corrosion (32.1% IACS).

Table 4 : Actual and estimated layer thickness (top depression).

Layer	Material	Actual	Inversion estimate	
		Thickness / mm	Course / mm	Fine / mm
Top	Aluminum	3.81	4.0	3.9
Middle	Air	2.94	3.0	3.1
Bottom	Aluminum	6.63	6.0	6.5

Table 5 : Actual and estimated layer thickness (bottom depression).

Layer	Material	Actual	Inversion estimate	
		Thickness / mm	Course / mm	Fine / mm
Top	Aluminum	6.68	7.0	6.5
Middle	Air	2.87	3.5	2.3
Bottom	Aluminum	3.81	3.5	3.7

Table 6 : Actual and estimated layer thickness (plate separation).

Layer	Material	Actual	Inversion estimate	
		Thickness / mm	Course / mm	Fine / mm
Top	Aluminum	6.68	7.5	6.7
Middle	Air	0.84	1.5	0.9
Bottom	Aluminum	6.63	6.5	6.3

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